

2. Theoretical Motivation

2.1 MuCAP

Precision measurements of muon capture by the proton provide a challenging opportunity to test our understanding of chiral symmetry breaking in QCD. In the absence of second class currents, the electroweak structure of a nucleon can be described by four form factors G_V , G_M , G_A , and G_P that determine the matrix elements of the charged vector and axial currents:

$$\langle n|V_\mu^-|p\rangle = \bar{u}(p_n) \left(G_V(q^2)\gamma_\mu + i\frac{G_M(q^2)}{2M_N}\sigma_{\mu\nu}q^\nu \right) u(p_p) \quad (2.1)$$

$$\langle n|A_\mu^-|p\rangle = \bar{u}(p_n) \left(G_A(q^2)\gamma_\mu\gamma_5 + \frac{G_P(q^2)}{2M_N}q_\mu\gamma_5 \right) u(p_p) \quad (2.2)$$

where p_p and p_n are the four-momenta of the initial proton and the final neutron, $u(p_p)$ and $u(p_n)$ are the corresponding spinors, M_N is the nucleon mass, and $q = (p_n - p_p)$. Theoretically, the reaction $\mu + p \rightarrow \nu_\mu + n$ at rest is expected to be well described by effective theory methods, like baryon chiral perturbation theory [1, 2, 3, 4], since the characteristic momentum transfer $q^2 = q_0^2 = -0.88m_\mu^2$ is small. While the vector and magnetic form factors are well known from electron scattering data (see [6] and references therein):

$$g_V = G_V(q_0^2) = 0.9755 \pm 0.0005 \quad (2.3)$$

$$g_M = G_M(q_0^2) = 3.5821 \pm 0.0025 \quad , \quad (2.4)$$

and the axial form factor can be calculated from the nucleon axial coupling constant $g_A = 1.2670 \pm 0.0035$ [7] and from the nucleon axial charge radius $r_A = (0.65 \pm 0.03)$ fm [1]:

$$G_A(q_0^2) = 1.245 \pm 0.003 \quad , \quad (2.5)$$

the pseudoscalar form factor

$$g_P = \frac{m_\mu}{2M_N}G_P(q_0^2) \quad (2.6)$$

is experimentally known to much less precision.

The present controversial experimental situation in ordinary (OMC) and radiative (RMC) muon capture is summarized in Fig. 2.1. The recent RMC result $g_P = 12.2 \pm 0.9 \pm 0.4$ [8] exceeds the theoretical predictions

$$g_P = \begin{cases} 8.44 \pm 0.23 & [1] \\ 8.21 \pm 0.09 & [2] \end{cases} \quad (2.7)$$

by 4σ . The older OMC results, on the other hand, had the difficult problem of absolute calibration of the neutron detection efficiency.

This was the main reason why they provided measurements of Λ_s at the 10% level only. The most accurate measurement with 4.5% precision was performed in Saclay using the lifetime technique in a liquid hydrogen target. At this high density μp capture proceeds not only from the free proton, but

also from the ortho and para states of the $p\mu p$ molecule. As indicated in Fig. 2.1 the uncertainty in the transition rate λ_{op} between these states leads to a significantly larger error in the interpretation of this measurement.

Our recent experiment on $\mu^3\text{He}$ capture [9] gives $g_p = 8.53 \pm 1.54$ (the accuracy is limited by the theoretical extraction of g_P from the three-nucleon system) in a better agreement with the theoretical values (2.7).

Theoretically, the dominant contribution to the pseudoscalar form factor is given by the pion pole (PCAC), and the leading correction to the pole term can be derived from QCD Ward identities [1] confirming the old current-algebra result [10]

$$G_P(q^2) = \frac{4M_N g_{\pi NN} F_\pi}{m_\pi^2 - q^2} - \frac{2}{3} g_A(0) M_N^2 r_A^2 \quad (2.8)$$

where $g_{\pi NN}$ is the pion-nucleon coupling constant, and F_π is the pion decay constant.

Possible corrections to Eq.(2.8) appear to be small [3, 4], however, further theoretical studies are needed in order to conclude whether the relation (2.8) can be eventually used to determine the πNN coupling constant $g_{\pi NN}$ from a precise measurement of g_P .

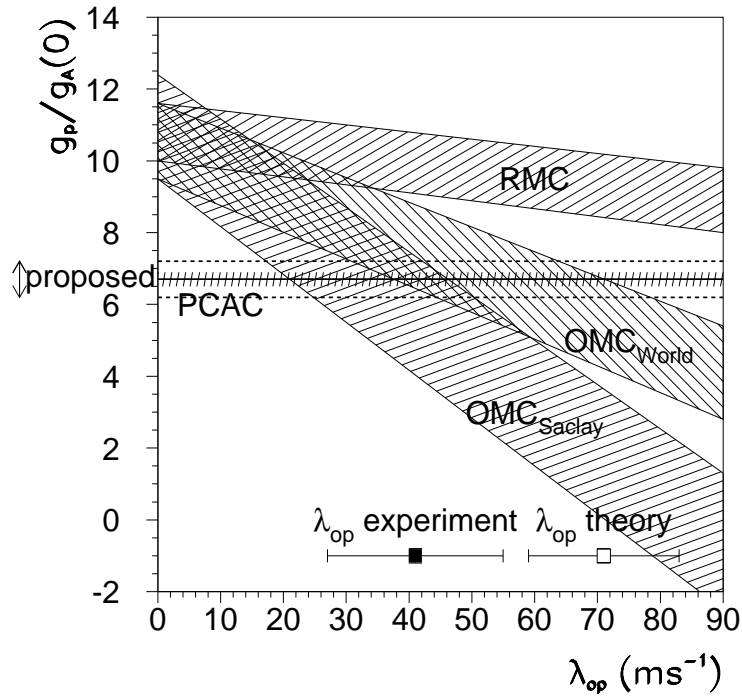


Figure 2.1: Current constraints on g_P as function of the ortho-para transition rate λ_{OP} . Experimental results from ordinary muon capture (OMC) [11], radiative muon capture (RMC) [8], and theory (PCAC).

The theoretical calculations of muon capture by the proton were recently reanalyzed in [12], with great care taken to include all known effects and keep approximations to the strictest minimum. The predicted singlet capture rate

$$\Lambda_s = 688.4 \pm 3.8 \text{ s}^{-1} \quad (2.9)$$

corresponds to $g_P = 8.475 \pm 0.076$ calculated with the pion-nucleon coupling constant $g_{\pi NN} = 13.37 \pm 0.09$ [13].

A significant progress in the understanding of the muon capture in the framework of the low energy effective theory of QCD was made recently by two groups [4, 14]. The results of the heavy-baryon chiral

Reference	[12]	[4]	[14]
$\Lambda_s \text{ s}^{-1}$	688.4 ± 3.8	687.4	695

Table 2.1: Results of recent calculations of the singlet capture rate Λ_s

perturbation theory (HBChPT) [14] and the small-scale expansion [4] are in a good agreement with the analysis [12] as shown in Table 2.1.

From the measured singlet capture rate Λ_s one can determine a linear combination of the axial and pseudoscalar form factors in a model independent way, as demonstrated in Fig. 2.2. The ultimate goal of the present experiment is to achieve an accuracy in the Λ_s measurement comparable with the present theoretical uncertainties in g_P .

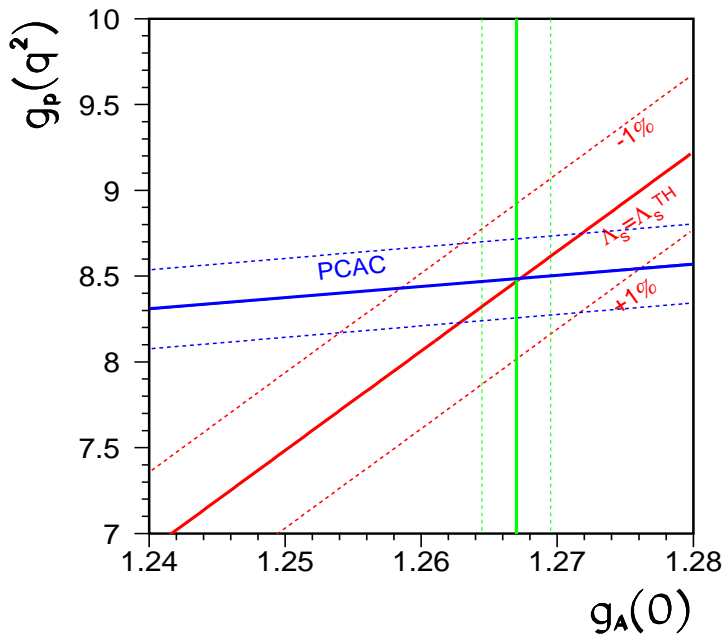


Figure 2.2: The contour plot of the singlet capture rate Λ_s as a function of the axial charge $g_A(0)$ and the pseudoscalar form factor $g_P(q^2)$. The bands show the PCAC relation and the present value of $g_A(0)$.

2.2 MuLAN

Theoretical Motivation

The Fermi Coupling Constant G_F is one of the few fundamental constants of nature. As such one should constantly strive to empirically determine it with the highest precision consistent with technological advances.

The Standard Model of Electroweak Interactions has three parameters (not counting M_H and the fermion masses and mixings). The choice of free parameters, which uses the most precise experimental values, is α , M_Z and G_F . Their experimental values, along with their absolute and relative errors are given by PDG2000 [7] as

$$\begin{aligned}
\alpha &= 1/137.03599976(50) && (0.037 \text{ ppm}) \\
M_Z &= 91.1882(22) \text{ GeV} && (23 \text{ ppm}) \\
G_F &= 1.16639(1) \times 10^{-5} \text{ GeV}^{-2} && (9 \text{ ppm})
\end{aligned} \tag{2.10}$$